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MASSIVE-QUARK BARYONS AS SKYRMIONS

Magnetic moments

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Abstract: The magnetic moments of the massive-quark baryons – strange and charmed hyperons – are calculated in the skyrmion description in which the baryon with a heavy flavor is described as a heavy pseudoscalar meson Φ_Q composed of a heavy quark Q and a light antiquark $\bar{q}$ "wrapped" by – and bound to – an SU(2) soliton. We use the original Skyrme-type lagrangian supplemented by a symmetry breaking term involving derivatives of the chiral field U. Both the spectra and the magnetic moments predicted by this model are quite similar to those of quark models. Our results provide evidence that the skyrmion description works equally well for massive-quark baryons as it does for light-quark (chiral-symmetry) baryons, supporting the suggestion that a hierarchy of induced gauge fields associated with layers of length scales involved in the strong interactions play an important role.

1. Introduction

The bound soliton-pseudoscalar doublet model proposed by Callan and Klebanov [1]) has been found to work reasonably well in describing the structure of strangeness-flavored hyperons such as mass spectrum [2–5]) and the magnetic moments [6–8]). The key feature of the model is that a pseudoscalar doublet meson Φ_Q made of a massive quark Q and a light antiquark $\bar{q}$ gets wrapped by – and bound to – an SU(2) soliton to give rise to the hyperons of one or more Q's. It was suggested recently [9]) that this picture should apply equally well to charmed and bottom baryons. This suggestion was supported by the hyperfine splittings in the heavy-flavor spectra but the centroid of each massive flavor came out too low because of the too strong binding of the Φ_Q with the soliton. This difficulty was resolved by Riska and Scoccola [10]) by adding to the usual Skyrme model an additional flavor symmetry breaking term that depends upon derivatives of the chiral field U, a term recently studied in a different context by Pari *et al.* [11]). The Riska–Scoccola model (called the RS model in short) differs

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from the usual Skyrme model in that in addition to the symmetry breaking in the Φ_Q mass (from, say, the pion mass), the effect on the decay constants is also taken into account through the derivative-dependent symmetry breaking. This model has been recently used to predict the spectra of both ground state and excited charmed and bottom baryons [12].

The aim of this paper is to provide a stronger case for the thesis of ref. [9] by calculating the magnetic moments of both strange- and charm-flavored baryons in the RS model. We confirm the results of ref. [10] in the spectra and obtain the hyperon magnetic moments in a *parameter-free* manner which agree surprisingly well with experiments and/or quark model results. What transpires from these results is that not only is the skyrmion description viable in the heavy-quark sector but also it provides us a deep insight into the working of induced gauge structure proposed in ref. [9].

2. The model

In this paper, we study the usual Skyrme model that consists of the quadratic current algebra term plus the Skyrme quartic term supplemented by symmetry-breaking terms. Implementation of vector mesons as in refs. [4,8] would bring in additional improvements, and so the results reported here can be considered as something that can be definitely improved upon. We start with the effective action for the simple Skyrme model with an appropriate symmetry breaking, expressed in terms of the SU(3)-valued chiral field $U(x)$ as

$$\Gamma = \int_{M_4} \mathrm{d}^4x \left\{ \tfrac{1}{16}F_\pi^2 \,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2}\mathrm{Tr}\left[[U^\dagger\partial_\mu U, U^\dagger \partial_\nu U]^2\right] \right\} + \Gamma_{\mathrm{WZ}} + \Gamma_{\mathrm{SB}}, \tag{1}$$

where F_π is the pion decay constant (= 186 MeV empirically), e is the so-called Skyrme parameter and M_4 denotes the (3 + 1)-dimensional spacetime manifold. In eq. (1), Γ_{WZ} is the Wess–Zumino action

$$\Gamma_{\mathrm{WZ}} = -\frac{iN_c}{240\pi^2}\int_{M_4\times[0,1]} \mathrm{Tr}[(U^\dagger dU)^5], \tag{2}$$

where N_c is the number of colors (= 3 in nature) and Γ_{SB} is responsible for the explicit symmetry breaking of chiral symmetry. This symmetry breaking is partially due to the finite mass of the pseudoscalar mesons. In the SU(3) case this effect can be taken into account by [13,14]

$$\Gamma_{\mathrm{SB}}^{\mathrm{mass}} = \int_{M_4} \mathrm{d}^4x \left\{ \tfrac{1}{48}F_\pi^2\left(m_\pi^2 + 2\, m_{\mathrm{K}}^2\right)\mathrm{Tr}\left[U + U^\dagger - 2\right] \right.$$
$$\left. + \tfrac{1}{24}\sqrt{3}F_\pi^2\left(m_\pi^2 - m_{\mathrm{K}}^2\right)\mathrm{Tr}\left[\lambda^8\left(U + U^\dagger\right)\right]\right\}, \tag{3}$$

where λ^8 is the eighth Gell-Mann matrix* and m_π and m_{K} represent the pion and kaon masses, respectively. The symmetry breaking term eq. (3) takes care of the mass

* We use the normalization $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$.

difference $m_K > m_\pi$ but not necessarily, at least to tree order, of other flavor-symmetry-breaking effects such as that $F_K \neq F_\pi$. The main effect of failing to account for the latter is that the kaon is overbound to the soliton [2,4]): This overbinding becomes more serious, the heavier the Φ_Q is. Riska and Scoccola [10]) have indeed shown that this defect can be mostly eliminated if the difference in the decay constants is properly taken into account in the lagrangian. Following their procedure, we introduce an additional symmetry-breaking lagrangian [11]) which naturally arises in chiral perturbation theory [15])

$$\delta\Gamma_{SB} = \frac{F_K^2 - F_\pi^2}{48} \int_{M_4} d^4x \, \mathrm{Tr}\left[\left(1 - \sqrt{3}\lambda_8\right)\left\{2\, m_K^2 \left(U + U^\dagger - 2\right) + \left(U\partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger\right)\right\}\right]. \tag{4}$$

Therefore, our total Γ_{SB} is given by

$$\Gamma_{SB} = \Gamma_{SB}^{\mathrm{mass}} + \delta\Gamma_{SB}. \tag{5}$$

We continue by introducing the Callan–Klebanov (CK) ansatz for the chiral field [1])

$$U_{CK} = \sqrt{U_\pi}\, U_3 \sqrt{U_\pi}, \tag{6}$$

$$\sqrt{U_\pi} = \begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix}, \qquad U_3 = \exp\left[\frac{i2\sqrt{2}}{F_\pi}\begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}\right], \tag{7}$$

$$N = \exp\left[\frac{i}{F_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right], \qquad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}. \tag{8}$$

Inserting eq. (6) into eq. (1) and expanding to second order in kaon fields, we can obtain the lagrangian density of the system. However, to recover the canonical form of the free kaon lagrangian when the interaction with the soliton is turned off, it is convenient to renormalize the kaon field K as K/χ, where χ is

$$\chi = \frac{F_K}{F_\pi}. \tag{9}$$

This leads to the final form of our kaon–soliton effective lagrangian, which reads

$$\mathcal{L} = \mathcal{L}_{SU(2)} + \mathcal{L}_K, \tag{10}$$

$$\mathcal{L}_{SU(2)} = \tfrac{1}{16}F_\pi^2 \,\mathrm{Tr}\left(\partial_\mu U_\pi^\dagger \partial^\mu U_\pi\right) + \frac{1}{32e^2}\mathrm{Tr}\left[\partial_\mu U_\pi^\dagger U_\pi, \partial_\nu U_\pi^\dagger U_\pi\right]^2 + \tfrac{1}{16}F_\pi^2 m_\pi^2 \,\mathrm{Tr}\left(U_\pi + U_\pi^\dagger - 2\right), \tag{11}$$

$$\begin{aligned}\mathcal{L}_K = {} & (D_\mu K)^\dagger D^\mu K - K^\dagger \alpha_\mu^\dagger \alpha^\mu K - m_K^2 K^\dagger K - \tfrac{1}{4}m_\pi^2 \frac{1}{\chi^2} K^\dagger \left(U_\pi + U_\pi^\dagger - 2\right) K \\ & - \frac{1}{8e^2 F_\pi^2}\frac{1}{\chi^2} K^\dagger K \,\mathrm{Tr}\left[\partial_\mu U_\pi^\dagger U_\pi, \partial_\nu U_\pi^\dagger U_\pi\right]^2 \\ & - \frac{1}{e^2 F_\pi^2}\frac{1}{\chi^2}\left\{2(D_\mu K)^\dagger D_\nu K \,\mathrm{Tr}(\alpha^\mu \alpha^\nu) + \tfrac{1}{2}(D_\mu K)^\dagger D^\mu K \,\mathrm{Tr}\left(\partial_\nu U_\pi^\dagger \partial^\nu U_\pi\right)\right. \\ & \left. -6(D_\mu K)^\dagger \left[\alpha^\mu, \alpha^\nu\right] D_\nu K\right\} - \frac{iN_c}{F_\pi^2}\frac{1}{\chi^2} B^\mu \left[K^\dagger D_\mu K - (D_\mu K)^\dagger K\right],\end{aligned} \tag{12}$$

where

$$D_\mu = \partial_\mu + v_\mu ,$$
$$\begin{pmatrix} v_\mu \\ \alpha_\mu \end{pmatrix} = \tfrac{1}{2}\left(N^\dagger \partial_\mu N \pm N \partial_\mu N^\dagger\right) \tag{13}$$

and B^μ is the baryon current

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\beta\eta}\,\mathrm{Tr}\left[U_\pi^\dagger\,(\partial_\nu U_\pi)\,U_\pi^\dagger\,(\partial_\beta U_\pi)\,U_\pi^\dagger\,(\partial_\eta U_\pi)\right] . \tag{14}$$

In comparing eq. (12) with the corresponding kaon lagrangian used in refs. [1,2]) (where pions were taken massless), it is clear that the net effect of introducing the extra symmetry breaking term $\delta\Gamma_{\mathrm{SB}}$ is the reduction of the contributions from the Wess–Zumino term and the Skyrme quartic term. Since the kaon binding energy is mainly determined by the Wess–Zumino term, its decrease by a factor of $1/\chi^2$ (for $\chi > 1$) immediately leads to a smaller binding energy which goes in the right direction to improve the $\mathrm{O}(N_c^0)$ prediction of the model.

Following the standard procedure, we use the hedgehog ansatz

$$N = \exp\left[\tfrac{1}{2} i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)\right] \tag{15}$$

to determine the soliton properties. The profile function $F(r)$ is of course obtained by minimizing the soliton energy. Given the soliton profile, one can proceed to solve the eigenvalue equation of kaons moving in the background potential provided by the soliton. This determines the kaon energy ω which is of $\mathrm{O}(N_c^0)$ in N_c counting and its wavefunction $k(r)$. Finally, to obtain the (hyperfine) splitting between states with same strange quantum number but different spin–isospin quantum numbers, the soliton has to be rotated in the SU(2) isospace. This provides the $\mathrm{O}(1/N_c)$ contribution to the mass. Details of this procedure which can be found in refs. [2,4]) will be omitted here.

So far, we discussed the procedure for strange hyperons. As proposed first in ref. [9]), charmed baryons can be described in the present model by formally extending the field U to the SU(4) group*. The generalized Callan–Klebanov ansatz can then be written as

$$U = \sqrt{U_\pi}\,U_4\sqrt{U_\pi} , \tag{16}$$

where U_π represents the SU(2) soliton field. The explicit form of U_π is

$$U_\pi = \begin{pmatrix} N^2 & 0 \\ 0 & \mathbf{1}_2 \end{pmatrix} , \tag{17}$$

where $\mathbf{1}_2$ is the 2×2 unit matrix. For U_4, we write

$$U_4 = \exp\left\{ i\frac{2\sqrt{2}}{F_\pi}\begin{pmatrix} \mathbf{0}_2 & K & D \\ K^\dagger & 0 & 0 \\ D^\dagger & 0 & 0 \end{pmatrix}\right\} \tag{18}$$

* It should be stressed that we are *not* assuming a symmetry group here. It is just a convenience in organizing the relevant degrees of freedom and can be easily avoided, as discussed in ref. [16]). To the extent that we limit ourselves to quadratic order in Φ_{Q} field, the two procedures are totally equivalent.

with $\mathbf{0}_2$ the 2×2 null matrix. Here, K and D represent the K-meson and the D-meson doublets, (so far generally denoted as Φ_Q), defined as

$$K = \frac{1}{\chi_1}\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad D = \frac{1}{\chi_2}\begin{pmatrix} \overline{D}^0 \\ D^- \end{pmatrix}, \tag{19}$$

where we have introduced different meson decay constant ratios χ_i for different flavored mesons. Note that in eq. (18), the pseudoscalar $D_s^\pm$ mesons composed of S- and C-quarks are not included to be consistent with the quadratic approximation that we make in the meson fields [9]).

The hamiltonian as well as the equations of motion for K- and D-mesons are obtained from the lagrangian of eq. (12) with eqs. (17) and (18). Since interactions between the K- and D-mesons can be ignored within the quadratic approximation, the equations of motion for K and D are formally identical, the only differences being in the meson masses and the constants χ's. We take the experimental meson masses, $m_K = 495$ MeV and $m_D = 1867$ MeV. The values for the meson decay constant ratios χ_i will be given later.

The mass formula for the baryons is

$$M(I, J, n_1, n_2, J_1, J_2, J_m) = M_{\rm sol} + n_1\omega_1 + n_2\omega_2 + M_{\rm rot}, \tag{20}$$

$$\begin{aligned} M_{\rm rot} = \frac{1}{2\mathcal{I}}\Bigg\{ & I(I+1) + (c_1 - c_2)[c_1 J_1(J_1+1) \\ & - c_2 J_2(J_2+1)] + c_1 c_2 J_m(J_m+1) \\ & + [J(J+1) - J_m(J_m+1) - I(I+1)] \\ & \times \left[\frac{c_1+c_2}{2} + \frac{c_1-c_2}{2}\frac{J_1(J_1+1) - J_2(J_2+1)}{J_m(J_m+1)}\right]\Bigg\}, \end{aligned} \tag{21}$$

where $M_{\rm sol}$ is the soliton mass and $\mathcal{I}$ the SU(2) moment of inertia. Here n_1 is the absolute value of strangeness, n_2 the charm quantum number and ω_1 and ω_2 are, respectively, the bound-state energies of the K- and D-mesons. In addition, c_1 is the hyperfine splitting constant corresponding to K and c_2 the one corresponding to D. For completeness, the explicit expressions of $M_{\rm sol}$, $\mathcal{I}$ and c_i are given in appendix A.

The angular momenta J_1 and J_2 are defined as $J_i = n_i j_i$ with j_i standing for the angular momentum of the bound-state orbital ($j_i = \frac{1}{2}$ for the lowest-energy state in which we are interested here) and J_m is given by $J_m = J_1 + J_2, \ldots, |J_1 - J_2|$. J is the total angular momentum

$$\boldsymbol{J} = \boldsymbol{R} + \boldsymbol{J}_m, \tag{22}$$

where $\boldsymbol{R}$ is the rotor spin. Only J is a good quantum number: Neither R nor J_m is separately conserved. Within our scheme, the quantum numbers of the physical hyperons can be obtained by using the quantization rules described in ref. [17]). They are

TABLE 1
Even-parity $J = \frac{1}{2}$ and $\frac{3}{2}$ baryons. The states of Ξ_c and Ξ_c' are mixed states of $J_m = 0$ and 1. See table 2 and discussion in sect. 4.

Particle	I	J	S	C	R	J_1	J_2	J_m
N	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	0
Λ	0	$\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$
Σ	1	$\frac{1}{2}$	-1	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$
Σ^*	1	$\frac{3}{2}$	-1	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$
Ξ	$\frac{1}{2}$	$\frac{1}{2}$	-2	0	$\frac{1}{2}$	1	0	1
Ξ^*	$\frac{1}{2}$	$\frac{3}{2}$	-2	0	$\frac{1}{2}$	1	0	1
Ω	0	$\frac{3}{2}$	-3	0	0	$\frac{3}{2}$	0	$\frac{3}{2}$
Λ_c	0	$\frac{1}{2}$	0	1	0	0	$\frac{1}{2}$	$\frac{1}{2}$
Σ_c	1	$\frac{1}{2}$	0	1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
Σ_c^*	1	$\frac{3}{2}$	0	1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
Ξ_{cc}	$\frac{1}{2}$	$\frac{1}{2}$	0	2	$\frac{1}{2}$	0	1	1
Ξ_{cc}^*	$\frac{1}{2}$	$\frac{3}{2}$	0	2	$\frac{1}{2}$	0	1	1
Ω_{ccc}	0	$\frac{3}{2}$	0	3	0	0	$\frac{3}{2}$	$\frac{3}{2}$
Ξ_c	$\frac{1}{2}$	$\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1,0
Ξ_c'	$\frac{1}{2}$	$\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1,0
Ξ_c^*	$\frac{1}{2}$	$\frac{3}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Ω_c	0	$\frac{1}{2}$	-2	1	0	1	$\frac{1}{2}$	$\frac{1}{2}$
Ω_c^*	0	$\frac{3}{2}$	-2	1	0	1	$\frac{1}{2}$	$\frac{3}{2}$
Ω_{cc}	0	$\frac{1}{2}$	-1	2	0	$\frac{1}{2}$	1	$\frac{1}{2}$
Ω_{cc}^*	0	$\frac{3}{2}$	-1	2	0	$\frac{1}{2}$	1	$\frac{3}{2}$

summarized in table 1. Using table 1, we can easily read off the baryon wavefunctions. In table 2, the wavefunctions of spin-up baryons are given in the basis, i.e., $|\,I, I_z; S, C\rangle_I$ $|\,R, R_z\rangle_R\;|\;J_1, J_{1,z}\rangle_S\;|\;J_2, J_{2,z}\rangle_C$.

3. Magnetic moments

Given the electromagnetic current $J_\mu^{\text{e.m.}} = J_\mu^3 + \sqrt{1/3}J_\mu^8$ obtained from our effective lagrangian by means of Noether's theorem, the magnetic moment operator is of the standard form

$$\boldsymbol{\mu} = \tfrac{1}{2}\int \mathrm{d}^3x\, \boldsymbol{r} \times \boldsymbol{J}^{\text{e.m.}}. \tag{23}$$

A lengthy but straightforward calculation leads to the third component of $\boldsymbol{\mu}$ of the form

$$\mu^3 = \mu_s^3 + \mu_v^3, \tag{24}$$

TABLE 2
States of spin-up baryons

Particle	State
$\lvert p\rangle = \lvert \frac{1}{2},\frac{1}{2};0,0\rangle_I \lvert \frac{1}{2},\frac{1}{2}\rangle_R$	
$\lvert n\rangle = \lvert \frac{1}{2},-\frac{1}{2};0,0\rangle_I \lvert \frac{1}{2},\frac{1}{2}\rangle_R$	
$\lvert \Lambda^0\rangle = \lvert 0,0;-1,0\rangle_I \lvert 0,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S$	
$\lvert \Sigma^a\rangle = \lvert 1,a;-1,0\rangle_I \left\{\sqrt{\frac{2}{3}} \lvert 1,1\rangle_R \lvert \frac{1}{2},-\frac{1}{2}\rangle_S - \sqrt{\frac{1}{3}} \lvert 1,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S\right\}$	
$\lvert \Sigma^{*,a}\rangle = \lvert 1,a;-1,0\rangle_I \lvert 1,1\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S$	
$\lvert \Xi^a\rangle = \lvert \frac{1}{2},a+\frac{1}{2};-2,0\rangle_I \left\{-\sqrt{\frac{1}{3}} \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert 1,0\rangle_S + \sqrt{\frac{2}{3}} \lvert \frac{1}{2},-\frac{1}{2}\rangle_R \lvert 1,1\rangle_S\right\}$	
$\lvert \Xi^{*,a}\rangle = \lvert \frac{1}{2},a+\frac{1}{2};-2,0\rangle_I \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert 1,1\rangle_S$	
$\lvert \Omega^-\rangle = \lvert 0,0;-3,0\rangle_I \lvert 0,0\rangle_R \lvert \frac{3}{2},\frac{3}{2}\rangle_S$	
$\lvert \Lambda_c^+\rangle = \lvert 0,0;0,1\rangle_I \lvert 1,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_C$	
$\lvert \Sigma_c^a\rangle = \lvert 1,a-1;0,1\rangle_I \left\{\sqrt{\frac{2}{3}} \lvert 1,1\rangle_R \lvert \frac{1}{2},-\frac{1}{2}\rangle_C - \sqrt{\frac{1}{3}} \lvert 1,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_C\right\}$	
$\lvert \Sigma_c^{*,a}\rangle = \lvert 1,a-1;0,1\rangle_I \lvert 1,1\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_C$	
$\lvert \Xi_{cc}^a\rangle = \lvert \frac{1}{2},a-\frac{3}{2};0,2\rangle_I \left\{-\sqrt{\frac{1}{3}} \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert 1,0\rangle_C + \sqrt{\frac{2}{3}} \lvert \frac{1}{2},-\frac{1}{2}\rangle_R \lvert 1,1\rangle_C\right\}$	
$\lvert \Xi_{cc}^{*,a}\rangle = \lvert \frac{1}{2},a-\frac{3}{2};0,2\rangle_I \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert 1,1\rangle_C$	
$\lvert \Omega_{ccc}^{++}\rangle = \lvert 0,0;0,3\rangle_I \lvert 0,0\rangle_R \lvert \frac{3}{2},\frac{3}{2}\rangle_C$	
$\lvert \Xi_c^a\rangle = \lvert \frac{1}{2},a-\frac{1}{2};-1,1\rangle_I \left\{\sqrt{\frac{1}{2}} \lvert \frac{1}{2},-\frac{1}{2}\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C - \sqrt{\frac{1}{2}} \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert \frac{1}{2},-\frac{1}{2}\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C\right\}$	
$\lvert \Xi_c'^a\rangle = \lvert \frac{1}{2},a-\frac{1}{2};-1,1\rangle_I \left\{-\sqrt{\frac{1}{6}} \lvert \frac{1}{2},-\frac{1}{2}\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C -\sqrt{\frac{1}{6}} \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert \frac{1}{2},-\frac{1}{2}\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C + \sqrt{\frac{2}{3}} \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert \frac{1}{2},-\frac{1}{2}\rangle_C\right\}$	
$\lvert \Xi_c^{*,a}\rangle = \lvert \frac{1}{2},a-\frac{1}{2};-1,1\rangle_I \lvert \frac{1}{2},\frac{1}{2}\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C$	
$\lvert \Omega_c^0\rangle = \lvert 0,0;-2,1\rangle_I \left\{\sqrt{\frac{2}{3}} \lvert 0,0\rangle_R \lvert 1,1\rangle_S \lvert \frac{1}{2},-\frac{1}{2}\rangle_C - \sqrt{\frac{1}{3}} \lvert 0,0\rangle_R \lvert 1,0\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C\right\}$	
$\lvert \Omega_c^{*,0}\rangle = \lvert 0,0;-2,1\rangle_I \lvert 0,0\rangle_R \lvert 1,1\rangle_S \lvert \frac{1}{2},\frac{1}{2}\rangle_C$	
$\lvert \Omega_{cc}^+\rangle = \lvert 0,0;-1,2\rangle_I \left\{-\sqrt{\frac{1}{3}} \lvert 0,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert 1,0\rangle_C + \sqrt{\frac{2}{3}} \lvert 0,0\rangle_R \lvert \frac{1}{2},-\frac{1}{2}\rangle_S \lvert 1,1\rangle_C\right\}$	
$\lvert \Omega_{cc}^{*,+}\rangle = \lvert 0,0;-1,2\rangle_I \lvert 0,0\rangle_R \lvert \frac{1}{2},\frac{1}{2}\rangle_S \lvert 1,1\rangle_C$	

$$\mu_s^3 = \mu_{s,0} R^3 + \mu_{s,1} J_1^3 + \mu_{s,2} J_2^3 ,$$
$$\mu_v^3 = -2(\mu_{v,0} + \mu_{v,1}|S| + \mu_{v,2}|C|) D^{33} , \tag{25}$$

where S is the strangeness and C the charm number and D^{33} is $-I^3R^3/I(I+1)$. The $\mu_{s,i}$'s and $\mu_{v,i}$'s, expressed in units of Bohr magneton, are given by

$$\mu_{s,0} = -\frac{2M_N}{3\pi\mathcal{I}} \int dr\, r^2 \sin^2 F F' , \tag{26}$$
$$\mu_{s,1} = c_1 \mu_{s,0} - a(k,\chi_1) , \tag{27}$$
$$\mu_{s,2} = c_2 \mu_{s,0} + a(\Phi_D,\chi_2) , \tag{28}$$
$$\mu_{v,0} = \tfrac{1}{2} M_N \mathcal{I} , \tag{29}$$
$$\mu_{v,1} = b(k,\omega_1,\chi_1) , \tag{30}$$
$$\mu_{v,2} = b(\Phi_D,\omega_2,\chi_2) , \tag{31}$$

where

$$a(k,\chi_1) = \tfrac{4}{3} M_N \int dr\, r^2 \Big\{ k^2 \cos^2 \tfrac{1}{2}F + \frac{1}{e^2 F_\pi^2} \frac{1}{\chi_1^2} \left[4\frac{k^2}{r^2} \sin^2 F \cos^2 \tfrac{1}{2}F + k^2 F'^2 \cos^2 \tfrac{1}{2}F + 3kk'F' \sin F \right] \Big\} , \tag{32}$$

$$\begin{aligned} b(k,\omega_1,\chi_1) = \tfrac{1}{3} M_N \int dr\, r^2 \Big\{ & k^2 \cos^2 \tfrac{1}{2}F \left(1 - 4\sin^2 \tfrac{1}{2}F\right) \\ & + \frac{1}{e^2 F_\pi^2} \frac{1}{\chi_1^2} \Big\{ 4\frac{k^2}{r^2} \sin^2 F \cos^2 \tfrac{1}{2}F \left(3 - 8\sin^2 \tfrac{1}{2}F\right) \\ & + k^2 F'^2 \cos^2 \tfrac{1}{2}F \left(1 - 18\sin^2 \tfrac{1}{2}F\right) \\ & + 2k'^2 \sin^2 F + 3kk'F' \sin F \left(3 - 4\sin^2 \tfrac{1}{2}F\right) \Big\} \Big\} \\ & + \frac{N_c}{6} \frac{M_N}{F_\pi^2 \pi^2} \frac{\omega_1}{\chi_1^2} \int dr\, r^2 k^2 \sin^2 F F' , \end{aligned} \tag{33}$$

and we have denoted the D-meson field by Φ_D*. As is obvious from eqs. (26)–(31), the coefficients $\mu_{s,2}$ and $\mu_{v,2}$ can be obtained from the expressions for $\mu_{s,1}$ and $\mu_{v,1}$, respectively, by replacing the K-meson wavefunction, the eigenenergy ω_1, the hyperfine constant c_1 and the ratio χ_1 by the corresponding D-meson ones. It should be noted that the sign of $\mu_{s,2}$ is opposite to that of $\mu_{s,1}$. The reason for the sign change in $\mu_{s,2}$ is that the charm number, C, of the D-meson is $+1$, whereas the strangeness number, S, of the K-meson is -1.

The explicit formulas for the magnetic moment of each baryon are given, in terms of the coefficients $\mu_{s,i}$'s and $\mu_{v,i}$'s, in table 3.

* The last term in eq. (33) was missed by the authors of ref. [7]). This term which comes from the Wess–Zumino term plays an important role in the isovector moments $\mu_{v,1}$ and $\mu_{v,2}$.

TABLE 3
Magnetic moment formulae of baryons

Particle	Magnetic moment
$\mu(\mathrm{p})$	$= \frac{1}{2}\mu_{s,0} + \frac{2}{3}\mu_{v,0}$
$\mu(\mathrm{n})$	$= \frac{1}{2}\mu_{s,0} - \frac{2}{3}\mu_{v,0}$
$\mu(\Lambda^0)$	$= \frac{1}{2}\mu_{s,1}$
$\mu(\Sigma^+)$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,1} + \frac{2}{3}(\mu_{v,0} + \mu_{v,1})$
$\mu(\Sigma^0)$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,1}$
$\mu(\Sigma^-)$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,1} - \frac{2}{3}(\mu_{v,0} + \mu_{v,1})$
$\mu(\Sigma^{*,+})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,1} + (\mu_{v,0} + \mu_{v,1})$
$\mu(\Sigma^{*,0})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,1}$
$\mu(\Sigma^{*,-})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,1} - (\mu_{v,0} + \mu_{v,1})$
$\mu(\Xi^0)$	$= -\frac{1}{6}\mu_{s,0} + \frac{2}{3}\mu_{s,1} - \frac{2}{9}(\mu_{v,0} + 2\mu_{v,1})$
$\mu(\Xi^-)$	$= -\frac{1}{6}\mu_{s,0} + \frac{2}{3}\mu_{s,1} + \frac{2}{9}(\mu_{v,0} + 2\mu_{v,1})$
$\mu(\Xi^{*,0})$	$= \frac{1}{2}\mu_{s,0} + \mu_{s,1} + \frac{2}{3}(\mu_{v,0} + 2\mu_{v,1})$
$\mu(\Xi^{*,-})$	$= \frac{1}{2}\mu_{s,0} + \mu_{s,1} - \frac{2}{3}(\mu_{v,0} + 2\mu_{v,1})$
$\mu(\Omega^-)$	$= \frac{3}{2}\mu_{s,1}$
$\mu(\Lambda_c^+)$	$= \frac{1}{2}\mu_{s,2}$
$\mu(\Sigma_c^{++})$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,2} + \frac{2}{3}(\mu_{v,0} + \mu_{v,2})$
$\mu(\Sigma_c^+)$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,2}$
$\mu(\Sigma_c^0)$	$= \frac{2}{3}\mu_{s,0} - \frac{1}{6}\mu_{s,2} - \frac{2}{3}(\mu_{v,0} + \mu_{v,2})$
$\mu(\Sigma_c^{*,++})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,2} + (\mu_{v,0} + \mu_{v,2})$
$\mu(\Sigma_c^{*,+})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,2}$
$\mu(\Sigma_c^{*,0})$	$= \mu_{s,0} + \frac{1}{2}\mu_{s,2} - (\mu_{v,0} + \mu_{v,2})$
$\mu(\Xi_{cc}^{++})$	$= -\frac{1}{6}\mu_{s,0} + \frac{2}{3}\mu_{s,2} - \frac{2}{9}(\mu_{v,0} + 2\mu_{v,2})$
$\mu(\Xi_{cc}^+)$	$= -\frac{1}{6}\mu_{s,0} + \frac{2}{3}\mu_{s,2} + \frac{2}{9}(\mu_{v,0} + 2\mu_{v,2})$
$\mu(\Xi_{cc}^{*,++})$	$= \frac{1}{2}\mu_{s,0} + \mu_{s,2} + \frac{2}{3}(\mu_{v,0} + 2\mu_{v,2})$
$\mu(\Xi_{cc}^{*,+})$	$= \frac{1}{2}\mu_{s,0} + \mu_{s,2} - \frac{2}{3}(\mu_{v,0} + 2\mu_{v,2})$
$\mu(\Omega_{ccc}^{++})$	$= \frac{3}{2}\mu_{s,2}$
$\mu(\Xi_c^+)$	$= \frac{1}{2}\mu_{s,2}$
$\mu(\Xi_c^0)$	$= \frac{1}{2}\mu_{s,2}$
$\mu(\Xi_c'^+)$	$= \frac{1}{3}\mu_{s,0} + \frac{1}{3}\mu_{s,1} - \frac{1}{6}\mu_{s,2} + \frac{4}{9}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2})$
$\mu(\Xi_c'^0)$	$= \frac{1}{3}\mu_{s,0} + \frac{1}{3}\mu_{s,1} - \frac{1}{6}\mu_{s,2} - \frac{4}{9}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2})$
$\mu(\Xi_c^{*,+})$	$= \frac{1}{2}\mu_{s,0} + \frac{1}{2}\mu_{s,1} + \frac{1}{2}\mu_{s,2} + \frac{2}{3}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2})$
$\mu(\Xi_c^{*,0})$	$= \frac{1}{2}\mu_{s,0} + \frac{1}{2}\mu_{s,1} + \frac{1}{2}\mu_{s,2} - \frac{2}{3}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2})$
$\mu(\Omega_c^0)$	$= \frac{2}{3}\mu_{s,1} - \frac{1}{6}\mu_{s,2}$
$\mu(\Omega_c^{*,0})$	$= \mu_{s,1} + \frac{1}{2}\mu_{s,2}$
$\mu(\Omega_{cc}^+)$	$= -\frac{1}{6}\mu_{s,1} + \frac{2}{3}\mu_{s,2}$
$\mu(\Omega_{cc}^{*,+})$	$= \frac{1}{2}\mu_{s,1} + \mu_{s,2}$

4. Numerical results

The mass formula eq. (20) with (21), despite its opaque form, essentially reflects the *symmetry structure* of the model and hence is generic of the meson–soliton bound picture independently of dynamical details of the effective lagrangian. The dynamics is encoded in what we will call "mass parameters": $M_{\rm sol}$, $\mathcal{I}$, c_1, c_2, ω_1 and ω_2. It has been shown in ref. [9]) that when these quantities are determined by fitting the experimental masses of N, Δ, Λ, Σ, Λ_c and Σ_c, eq. (21) gives predictions for the hyperon masses which are in a remarkably good agreement with the existing empirical data and with quark model predictions for octet and decuplet baryons. We will refer to the values of these quantities so determined as "empirical" (i.e., between quotation marks to distinguish them from truly empirical quantities) and demand that our dynamical models *predict* these quantities*. The "empirical" values are [9])

$$
\begin{aligned}
M_{\rm sol} &= 866\ \text{MeV}, & \mathcal{I} &= 1.01\ \text{fm}, \\
\omega_1 &= 223\ \text{MeV}, & \omega_2 &= 1418\text{MeV}, \\
c_1 &= 0.604, & c_2 &= 0.140.
\end{aligned}
\tag{34}
$$

In our numerical calculation, we will consider two sets of parameters in the SU(2) sector. In one case, we consider the chiral limit in the SU(2) sector, $m_\pi = 0$, and fit F_π and e to reproduce the "empirical values" of $M_{\rm sol}$ and $\mathcal{I}$. This corresponds to the result of ref. [18])

$$F_\pi = 129\ \text{MeV}, \qquad e = 5.45. \tag{35}$$

The second set of parameters is obtained for $m_\pi = 138$ MeV. It corresponds to the result of ref. [19])

$$F_\pi = 108\ \text{MeV}, \qquad e = 4.84. \tag{36}$$

The predictions of the "mass parameters" in the strange and charm sectors for two sets of χ_i are given in table 4. In one case we set $\chi_{1,2} = 1$. This corresponds to switching off the extra symmetry breaking term $\delta\Gamma_{\rm SB}$ given in eq. (4). As mentioned before, flavored mesons are overbound to the soliton in this case. This effect is present for both massless and massive pions. When $\delta\Gamma_{\rm SB}$ is included, we use the empirical ratio $\chi_1 = 1.22$ in the strange sector. On the other hand, the empirical value of the ratio $\chi_2 = F_{\rm D}/F_\pi$ is not very well established. In the case of massless pion, we choose $\chi_2 = 1.8$ which falls well within the range given in ref. [20]), i.e., $F_{\rm D}/F_\pi = 1.8 \pm 0.2$. In the case of massive pion, we use a slightly larger value $\chi_2 = 2.0$ to obtain a better agreement with the "empirical values". In table 5 we show the baryon masses predicted

* In view of the fact that there is paucity of data in the charm sector, this procedure may not be as reliable as it is in the strange flavor sector. We will see later that our model with its predicted parameter values gives results in the charm sector which are in better overall agreement with quark-model results both in the spectra *and* in the magnetic moments than the "empirical" fit does.

TABLE 4

Mass formula parameters entering into eq. (21) calculated within our model. For the case of $m_\pi = 0$ we use the values of F_π and e given in eq. (35), while for $m_\pi = 138$ MeV we use those given in eq. (36). In both cases $M_{\rm sol} = 866$ MeV and $\mathcal{I} = 1.01$ fm.

	χ_1	ω_1 (MeV)	c_1	χ_2	ω_2 (MeV)	c_2
$m_\pi = 0$	1.00	153	0.62	1.00	760	0.16
	1.22	221	0.50	1.80	1303	0.21
$m_\pi \neq 0$	1.00	146	0.51	1.00	744	−0.02
	1.22	209	0.39	2.00	1342	0.13

TABLE 5

Baryon masses. Column 'Emp.' is calculated from the *"empirical"* values of eq. (34) as in ref. [9]. "SET I" means the results of $m_\pi = 0$, $F_\pi = 129$ MeV, $e = 5.45$, $\chi_1 = 1.22$ and $\chi_2 = 1.80$, "SET II" the results with $m_\pi = 138$ MeV, $F_\pi = 108$ MeV, $e = 4.84$, $\chi_1 = 1.22$, $\chi_2 = 2.00$. All values in MeV.

Particle	Exp.	Emp.	SET I	SET II	Ref. [22]	Ref. [23]
N	939	939	939	939		
Δ	1232	1232	1232	1232		
Λ	1116	1116	1106	1086		
Σ	1193	1193	1203	1205		
Σ^*	1385	1370	1350	1320		
Ξ	1318	1339	1332	1311		
Ξ^*	1530	1516	1480	1425		
Ω	1672	1669	1621	1549		
Λ_c	(2285)	2285	2172	2209	2200	2260
Σ_c	(2453)	2453	2327	2379	2360	2440
Σ_c^*	?	2494	2387	2417	2420	2510
Ξ_{cc}	?	3752	3513	3601	3550	
Ξ_{cc}^*	?	3793	3574	3639	3610	
Ω_{ccc}	?	5127	4791	4898	4810	
Ξ_c	(2470)	2499	2381	2426	2420	2480
Ξ_c'	?	2636	2509	2514	2523	2575
Ξ_c^*	?	2649	2524	2539	2531	2645
Ω_c	(2740)	2786	2643	2647	2680	2730
Ω_c^*	?	2811	2674	2662	2720	2790
Ω_{cc}	?	3939	3700	3764	3730	
Ω_{cc}^*	?	3964	3730	3778	3770	

by our model in comparison with the available experimental data [21]) and with the quark model predictions of refs. [22,23]). The "SET I" corresponds to

$$m_\pi = 0, \quad F_\pi = 129\,\mathrm{MeV}, \quad e = 5.45, \quad \chi_1 = 1.22, \quad \chi_2 = 1.8, \tag{37}$$

and the "SET II" to

$$m_\pi = 138\,\mathrm{MeV}, \quad F_\pi = 108\,\mathrm{MeV}, \quad e = 4.84, \quad \chi_1 = 1.22, \quad \chi_2 = 2.0. \tag{38}$$

We observe that our model, with or without pion mass, works very well for both strange and charmed baryons. In fact, the predictions of the model are *as a whole* in closer agreement with the quark model results for the charmed baryons than the "empirical" fits. This suggests that we should do an overall fit rather than fix the necessary parameters to the empirical values whose validity may be somewhat doubtful in the massive-quark sector. We apply this remark to magnetic moments discussed below. It is interesting to note that for "SET I" our results for the masses of Ξ_c and Ξ_c' differ slightly from those given in ref. [10]) where the same set of parameters was used. The reason for this difference is that here we use strange-charmed cascade wave functions which are linear combinations of those used in ref. [10]). We will come back later to this point which has a dramatic effect in the magnetic moments of these particles.

Before discussing our predictions for the baryon magnetic moments, we summarize the present status of the quark-model results available in the literature. Since experimental data are not yet available for charmed baryon magnetic moments, we will make comparison with quark model predictions. In fact in making a "model-independent" analysis of the sort we made for the masses, we will have to resort to quark-model predictions of the coefficients $\mu_{s,2}$ and $\mu_{v,2}$.

Choudhury and Joshi [24]) calculated the magnetic moments of charmed baryons, expressing them in terms of the proton and neutron magnetic moments via U(4) symmetry. Subsequently they used U(8) symmetry to express them all in terms of the proton magnetic moment, μ_p [ref. [25])]. Lichtenberg [26]) used a quark model implemented by the gauge structure of QCD considered by De Rújula, Georgi and Glashow [22]) and obtained results which differ significantly from those of Choudhury and Joshi. Only in the limit of equal quark masses do the results of ref. [26]) reduce to those of Choudhury and Joshi. The calculation of Jena and Rath [27]) of the magnetic moments of spin-$\frac{1}{2}$ charmed baryons in a relativistic logarithmic potential model is, on the other hand, in good agreement with that of Lichtenberg. We take this to mean that Lichtenberg's results are more reliable than those of Choudhury and Joshi. Furthermore, Bose and Singh used the MIT bag model [28]), obtaining results which are in a fair agreement with those of Lichtenberg. For instance, Lichtenberg's magnetic moment relation [our notation for the mixed ($S = -1$, $C = +1$) cascades differs from that of ref. [26]), see below.]

$$\mu(\Lambda_c^+) = \mu(\Xi_c^+) = \mu(\Xi_c^0) \tag{39}$$

holds well in the MIT bag model. In this work we will compare our results with those of both ref. [26]) and ref. [28]), keeping in mind that the present available quark-model results may not be fully realistic and hence may not describe nature accurately.

As in the case of the mass formula, the magnetic moment formula eq. (24) with eqs. (26)–(31) can be considered in a model-independent way: It reflects the symmetries of the model. Thus we may determine the coefficients $\mu_{s,i}$'s and $\mu_{v,i}$'s from experiments and/or quark-model results. The "magnetic moment parameters" so obtained will be referred to as "empirical". The dynamical content of a specific model can then be judged by the extent to which the model values agree with the "empirical" ones. The coefficients $\mu_{s,0}$, $\mu_{s,1}$, $\mu_{v,0}$ and $\mu_{v,1}$ are determined by fitting the magnetic moments of the proton, neutron and strange baryons to the experimental values by the least-square fitting*. There are no experimental data available to fix $\mu_{s,2}$ and $\mu_{v,2}$, so we will fix them to Lichtenberg's quark-model values for charmed baryons. The "magnetic moment parameters" so obtained are:

$$\begin{aligned} \mu_{s,0} &= 0.880, \quad & \mu_{s,1} &= -1.188, \quad & \mu_{s,2} &= 0.740, \\ \mu_{v,0} &= 3.530, \quad & \mu_{v,1} &= -0.934, \quad & \mu_{v,2} &= -0.695. \end{aligned} \tag{40}$$

In the case of massless pion the calculated coefficients in the SU(2) sector are [18])

$$\mu_{s,0} = 0.555, \qquad \mu_{v,0} = 2.402, \tag{41}$$

while for $m_\pi = 138$ MeV one obtains [19])

$$\mu_{s,0} = 0.735, \qquad \mu_{v,0} = 2.402. \tag{42}$$

Note that in both cases the calculated values are below what we call "empirical" magnetic moment parameters. This leads to a rather small value for the magnetic moment of the proton μ_p [refs. [18,19])]. Indeed, using the parameters eq. (41) and eq. (42) we obtain $\mu_p = 1.88$ and $\mu_p = 1.97$, respectively. These values should be compared with $\mu_p^{\rm exp} = 2.79$ [ref. [21])]. In contrast, the nonrelativistic quark model predicts [27]) $\mu_p^{\rm QM} = 2.79$ in close agreement with the empirical value while in the bag model one gets [29]) $\mu_p^{\rm BM} = 1.90$. On the other hand, it has been established that the soliton models predict the ratio μ_n/μ_p quite accurately. As we shall see, the ratios of the magnetic moments for both strange and charmed baryons do come out fairly well.

The predicted "magnetic moment parameters" for the same parameter sets as used to calculate the "mass formula parameters" are given in table 6. From tables 4 and 6, we observe that the kaon energy, ω_1, increases whereas the hyperfine constant c_1 decreases for an increasing χ_1. Therefore the isoscalar magnetic moment in the strangeness direction, $\mu_{s,1}$, is affected as it is closely related to the hyperfine constant. In contr the variation in χ does not modify significantly $\mu_{v,1}$. In the charmed sector, thin

* In refs. [7,8]), the authors determined the "empirical" moment parameters by fitting the moments of the proton, neutron, Λ^0 and Σ.

TABLE 6

Magnetic moment parameters entering in eq. (25) in the unit of Bohr magneton. The input parameters for the massless and massive pion cases are the same as in table 4.

	χ_1	$\mu_{s,1}$	$\mu_{v,1}$	χ_2	$\mu_{s,2}$	$\mu_{v,2}$
$m_\pi = 0$	1.00	−0.83	−0.12	1.00	0.61	−0.12
	1.22	−0.78	−0.13	1.80	0.40	−0.10
$m_\pi \neq 0$	1.00	−1.19	−0.14	1.00	0.63	−0.12
	1.22	−1.07	−0.16	2.00	0.40	−0.10

a bit different: Both the D-meson energy ω_2 and the hyperfine constant c_2 increase for an increased χ_2*. The absolute value of $\mu_{s,1}$ is small compared to the empirical moment parameter of eq. (40), but this can be enhanced by introducing vector mesons as indicated in ref. [8]). The dependence of both $\mu_{s,2}$ and $\mu_{v,2}$ on χ is quite similar to the strangeness case.

In comparing the values of $\mu_{v,1}$ for $\chi_1 = 1$ given in table 6 with those reported in ref. [7]) ($= -0.05$ for massless pions and -0.06 for $m_\pi = 138$ MeV) we notice that the effect of the last term in eq. (33) is to increase the absolute value of $\mu_{v,1}$ by more than a factor of 2. However, our calculated values are still much smaller than the "empirical" ones. We expect that the inclusion of other degrees of freedom, e.g. vector mesons, in our effective action can bring some additional improvement in the model predictions.

The calculated ratios of the baryon magnetic moments to that of the proton are given in table 7 in comparison with the existing empirical data [21]), quark model and bag model calculations. The magnetic moments given in table 7 can be obtained simply by putting into the formulas of table 3 the set of magnetic moment parameters shown in table 8. As mentioned above, the parameters are generally in good agreement with experiments and/or quark model results for the isoscalar moments even in heavy flavor sector. On the contrary, the values of $\mu_{v,1}$ and $\mu_{v,2}$ are smaller than the empirical ones by factor of $3 \sim 5$ in the both cases of massless and massive pions. This fact mainly causes the differences between the predictions of this model and of experiments (or quark model) for the non-zero isospin baryons.

Among the magnetic moments listed in table 7, those for the *cascades that contain one S-quark and one C-quark* require clarification. If one uses the wavefunctions of the model as calculated in ref. [9]) for such cascades, i.e., Ξ_c^+, Ξ_c^0, $\Xi_c'^+$ and $\Xi_c'^0$, then their magnetic moments take the following expressions

$$\mu(\Xi_c^+) = -\tfrac{1}{6}\mu_{s,0} + \tfrac{1}{3}\mu_{s,1} + \tfrac{1}{3}\mu_{s,2} - \tfrac{2}{9}\left(\mu_{v,0} + \mu_{v,1} + \mu_{v,2}\right) ,$$

* On general grounds, as discussed in ref. [16]), one expects c_2 to decrease for increasing ω_2. This suggests that something may be missing in the model. For the present system it does not seem too serious, so we shall not pursue this issue any further.

TABLE 7

Magnetic moments of baryons. SET I and SET II are the same as in table 5. "Quark model" stands for the results of ref. [26]) and "Bag model" for refs. [28,29]). All moments are given relative to the proton magnetic moment.

Particle	Exp.	SET I	SET II	Quark model	Bag model
p	1.00	1.00	1.00	1.00	1.00
n	−0.68	−0.70	−0.63	−0.67	−0.67
Λ^0	−0.22	−0.21	−0.27	−0.21	−0.25
Σ^+	0.87	1.07	1.10	0.96	0.97
Σ^0	–	0.27	0.34	0.29	0.31
Σ^-	−0.41	−0.54	−0.42	−0.38	−0.36
$\Sigma^{*,+}$	–	1.29	1.24	1.13	
$\Sigma^{*,0}$	–	0.09	0.10	0.13	
$\Sigma^{*,-}$	–	−1.12	−1.04	−0.87	
Ξ^0	−0.45	−0.58	−0.66	−0.50	−0.56
Ξ^-	−0.24	−0.07	−0.19	−0.16	−0.23
$\Xi^{*,0}$	–	0.49	0.35	0.25	
$\Xi^{*,-}$	–	−1.03	−1.07	−0.75	
Ω^-	–	−0.63	−0.82	−0.62	
Λ_c^+	–	0.11	0.10	0.13	0.18
Σ_c^{++}	–	0.98	0.99	0.85	0.70
Σ_c^+	–	0.16	0.21	0.18	0.13
Σ_c^0	–	−0.65	−0.56	−0.49	−0.44
$\Sigma_c^{*,++}$	–	1.62	1.64	1.47	1.40
$\Sigma_c^{*,+}$	–	0.40	0.47	0.47	0.48
$\Sigma_c^{*,0}$	–	−0.82	−0.69	−0.53	−0.43
Ξ_{cc}^{++}	–	−0.17	−0.17	−0.04	0.06
Ξ_{cc}^+	–	0.35	0.32	0.29	0.31
$\Xi_{cc}^{*,++}$	–	1.14	1.13	0.93	0.91
$\Xi_{cc}^{*,+}$	–	−0.42	−0.35	−0.07	0.07
Ω_{ccc}^{++}	–	0.32	0.30	0.40	0.52
Ξ_c^+	–	0.11	0.10	0.13	0.18
Ξ_c^0	–	0.11	0.10	0.13	0.18
$\Xi_c'^+$	–	0.44	0.39	0.26	0.17
$\Xi_c'^0$	–	−0.59	−0.57	−0.41	−0.39
$\Xi_c^{*,+}$	–	0.81	0.74	0.59	0.55
$\Xi_c^{*,0}$	–	−0.72	−0.71	−0.41	−0.36
Ω_c^0	–	−0.31	−0.40	−0.32	−0.35
$\Omega_c^{*,0}$	–	−0.31	−0.44	−0.28	−0.28
Ω_{cc}^+	–	0.21	0.23	0.25	0.30
$\Omega_{cc}^{*,+}$	–	0.01	−0.07	0.06	0.14

TABLE 8

Magnetic moment parameters relative to the proton magnetic moment. SET I, SET II and "Quark model" are the same as defined in table 7.

	$\mu_{s,0}$	$\mu_{s,1}$	$\mu_{s,2}$	$\mu_{v,0}$	$\mu_{v,1}$	$\mu_{v,2}$
Exp.	0.315	−0.425	-	1.264	−0.334	-
SET I	0.295	−0.417	0.213	1.278	−0.070	−0.055
SET II	0.373	−0.545	0.203	1.219	−0.079	−0.052
Quark model	0.333	−0.415	0.270	1.250	−0.250	−0.250

$$\mu(\Xi_c^0) = -\tfrac{1}{6}\mu_{s,0} + \tfrac{1}{3}\mu_{s,1} + \tfrac{1}{3}\mu_{s,2} + \tfrac{2}{9}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2}),$$
$$\mu(\Xi_c'^+) = \tfrac{1}{2}\mu_{s,0} + \tfrac{2}{3}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2}),$$
$$\mu(\Xi_c'^0) = \tfrac{1}{2}\mu_{s,0} - \tfrac{2}{3}(\mu_{v,0} + \mu_{v,1} + \mu_{v,2}). \tag{43}$$

These are significantly different from the corresponding formulas given in table 4*. Eq. (43) predicts

$$\mu(\Xi_c^+) = -0.72(-0.70,-0.82), \qquad \mu(\Xi_c^0) = 0.13(0.26,0.14),$$
$$\mu(\Xi_c'^+) = 1.71(1.72,1.80), \qquad \mu(\Xi_c'^0) = -0.83(-1.17,-1.06), \tag{44}$$

where the number outside of the parenthesis corresponds to using the "empirical" magnetic moment parameters, the first number inside the parenthesis to the calculated magnetic moment parameters for $m_\pi = 0$ and the second to the same for $m_\pi =$ 138 MeV. These should be compared with the quark-model results $\mu(\Xi_c^+) = 0.37$, $\mu(\Xi_c^0) = 0.37$, $\mu(\Xi_c'^+) = 0.73$ and $\mu(\Xi_c'^0) = -1.07$.

The difference between the two can be readily understood by noting that the two wavefunctions are related to each other by an orthogonal transformation. To see this, recall the angular momentum coupling of the mesons when two mesons of different species are involved. In refs. [9,10]), when there are two mesons of different flavors bound to the soliton, their spins are first coupled to $J_m = 0$ or 1 which is then coupled to the rotor angular momentum R to give the total spin. As the rotor wave function represents the contribution of the light flavor quark q, we will call this the $q(SC)$ coupling scheme. On the other hand, the quark-model wave functions for these mixed cascades are constructed in a different representation. There the S-quark is first coupled to the light quark q to give $J_m' = 0$ or 1 which is in turn coupled to the C-quark to give the total angular momentum. We call this the $(qS)C$ coupling scheme. Clearly the bound-state model wave function of ref. [9]) is a linear

* The use of different wave functions also affects the predicted values of the Ξ_c and Ξ_c' masses. However the modified values (e.g., 2540 MeV for Ξ_c and 2596 MeV for Ξ_c' with the "empirical" mass parameters) are not so different form those given in table 5.

combination of the quark wave functions, the relation being given by recoupling (Racah) coefficients*.

Strictly speaking, in the bound state model the baryon wave functions are defined to $O(N_c^0)$. At this order, both cascade states are degenerate in energy. This degeneracy is lifted at $O(N_c^{-1})$ when rotational corrections are included. However, within our scheme, this correction is treated only in first-order perturbation theory and consequently we cannot distinguish between the different linear combinations. There is of course the possibility of diagonalizing the rotational hamiltonian in the subspace of the mixed cascades. However, this would require going beyond $O(N_c^{-1})$ which would be inconsistent since effects of order higher than $O(N_c^{-1})$ have been systematically ignored in the calculation. Therefore – and in order to make a meaningful comparison with the quark-model results – for the mixed cascades, we perform a linear combination to give the quark-model representation. The resulting magnetic moments (for the "empirical" magnetic moment parameters) are

$$\begin{aligned} \mu(\Xi_c^+) &= 0.37(0.37), \qquad \mu(\Xi_c'^+) = 0.62(0.73), \\ \mu(\Xi_c^0) &= 0.37(0.37), \qquad \mu(\Xi_c'^0) = -1.07(-1.13), \end{aligned} \tag{45}$$

to be compared with Lichtenberg's quark model values in the parenthesis. We now see that the agreement is equally good in the mixed cascade sector and that the magnetic moment relation of eq. (39) holds as well. The results given in table 5 and 7 correspond to this combination of the wavefunctions**.

An interesting observation to make here is that the magnetic moments of the mixed cascades are very sensitive to the wavefunctions. For instance, suppose we take our hamiltonian truncated with no K–D interactions and diagonalize it exactly for the mixed cascades (although it implies going beyond $O(N_c^{-1})$ as already mentioned). Then we obtain

$$\mu(\Xi_c^+) = -0.50, \qquad \mu(\Xi_c'^+) = 1.49, \qquad \mu(\Xi_c^0) = -0.67, \qquad \mu(\Xi_c'^0) = -0.03.$$

Although the mixing is small, the effect on the magnetic moments is substantial. Implications of this sensitivity to the wave functions will be discussed below.

5. Conclusions

It is shown in this paper that the skyrmion description works equally well for massive-quark baryons as it does for light-quark systems. The effective lagrangian used

* There seems to be some confusion about the symbols used to denote the mixed cascades. As in ref. [30]), throughout this paper, we use the symbol Ξ_c for the state with the lower mass and Ξ_c' for the higher mass. In the $(qS)C$ basis this corresponds to using the symbol Ξ_c for the state in which q and S are in the antisymmetric configuration (previously called the A state) and Ξ_c' for the one in which they are in the symmetric configuration (previously called the S state). Note that our notation differs from that of refs. [26,27]).

** If quark-model wavefunctions are recoupled in the representation corresponding to the wavefunctions of ref. [9]), they give, $\mu(\Xi_c^+) = -0.76$, $\mu(\Xi_c^0) = 0.17$, $\mu(\Xi_c'^+) = 1.85$ and $\mu(\Xi_c'^0) = -0.93$, close to what we get in our model.

was the original Skyrme model which consists of the usual current algebra term plus the quartic Skyrme term supplemented by symmetry-breaking terms that account for masses and decay constants of the pseudoscalar meson doublets Φ_Q containing the massive quark Q. Implementation of vector meson degrees of freedom is expected to improve even more on the predictivity of the model.

It is particularly noteworthy that with only two parameters needed for light-quark (up and down) systems, and masses and decay constants taken from empirical sources, the model is able to fit not only the masses but also the magnetic moments of strange and charmed baryons. The agreement with experiments and quark-model predictions is quite remarkable and suggests strongly that the model is close to nature in its physics content. At first sight, this is surprising since the skyrmion model looks so different from the quark description. The crucial feature of the model is that a massive scalar doublet carrying the flavor quantum number of the massive quark gets bound to and wrapped by the SU(2) soliton, the quantum numbers arising through topologically induced transmutation, much as what happens to a scalar doublet in the presence of a 't Hooft–Polyakov monopole and to diatomic molecules with electrons coupled to slowly rotating diatoms [16]). The dynamics of the model is encoded in the Wess–Zumino term which controls essentially both fine and hyperfine structure splittings. As discussed in ref. [16]), the essential dynamics can be understood in terms of a hierarchy of induced gauge (Berry) connections generated in integrating out layers of length scales.

One potentially important difference between the soliton model discussed here and the quark models is that while masses are insensitive, the magnetic moments for the mixed cascades (and *only* for the mixed cascades) are quite sensitive to the mixing between different flavor components ignored in the model. The wave functions used in refs. [9,10]) lead to magnetic moment predictions for the mixed cascades that are markedly different from those of the quark models. It should however be noted that in our model, within the approximations we make, any linear combination of the mixed cascades will give the same energy to leading order and hence cannot be distinguished by energy considerations alone. Therefore in comparing with the quark-model results, we are allowed to rewrite the wave functions of the soliton model in the representation used in quark models. This is what we have done for the results given in table 7. The higher-order terms thus far ignored in the model will certainly lift the degeneracy and could give us unique wavefunctions. Note that the situation is similar in quark models [31]). Experimental data will eventually tell us which pictures are closer to nature. It may well be that none of the two schemes is correct.

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Appendix A

In this appendix we write down the explicit expressions of the quantities M_{sol}, $\mathcal{I}$ and the hyperfine splitting constants c_i appearing in eq. (20). They have been derived elsewhere.

The soliton mass M_{sol} is given by the expression

$$M_{sol} = 4\pi \int dr\, r^2 \left\{ \frac{F_\pi^2}{8} \left[F'^2 + 2\frac{\sin^2 F}{r^2} \right] + \frac{1}{2e^2} \frac{\sin^2 F}{r^2} \left[\frac{\sin^2 F}{r^2} + 2F'^2 \right] + \tfrac{1}{4} m_\pi^2 F_\pi^2 (1 - \cos F) \right\}, \tag{A.1}$$

and the SU(2) moment of inertia $\mathcal{I}$ by

$$\mathcal{I} = \tfrac{2}{3}\pi F_\pi^2 \int dr\, r^2 \sin^2 F \left(1 + \frac{4}{e^2 F_\pi^2} \left(F'^2 + \frac{\sin^2 F}{r^2} \right) \right). \tag{A.2}$$

In addition, eq. (20) contains the hyperfine splitting constants c_i. The c_i for K and D are formally identical, so we quote only the kaon sector

$$c_1 = 1 - 2\omega_1 \int dr\, k^2 \left\{ \tfrac{4}{3} f r^2 \cos^2 \tfrac{1}{2}F - \frac{2}{e^2 F_\pi^2 \chi_1^2} \left[\frac{d}{dr} \left(r^2 \sin F F' \right) - \tfrac{4}{3} \sin^2 F \cos^2 \tfrac{1}{2}F \right] \right\}, \tag{A.3}$$

with the radial function f defined as

$$f = 1 + \frac{1}{e^2 F_\pi^2} \frac{1}{\chi_1^2} \left[F'^2 + 2\frac{\sin^2 F}{r^2} \right], \tag{A.4}$$

and the kaon wave functions subject to the normalization condition

$$2 \int dr\, r^2 k^2 (f\omega_1 + \lambda) = 1, \tag{A.5}$$

where the radial function λ is given by

$$\lambda = -\frac{N_c}{2\pi^2 F_\pi^2} \frac{1}{\chi_1^2} \frac{\sin^2 F}{r^2} F'. \tag{A.6}$$

References

1) C.G. Callan and I. Klebanov, Nucl. Phys. **B262** (1985) 365
2) C.G. Callan, K. Hornbostel and I. Klebanov, Phys. Lett. **B202** (1988) 269
3) N.N. Scoccola, H. Nadeau, M.A. Nowak and M. Rho, Phys. Lett. **B201** (1988) 425
4) N.N. Scoccola, D.P. Min, H. Nadeau and M. Rho, Nucl. Phys. **A505** (1989) 497
5) U. Blom, K. Dannbom and D.O. Riska, Nucl. Phys. **A493** (1989) 384
6) E.M. Nyman and D.O. Riska, Nucl. Phys. **B325** (1989) 593
7) J. Kunz and P.J. Mulders, Phys. Lett. **B231** (1989) 335 ; Phys. Rev. **D41** (1990) 1578
8) D.P. Min, Y.S. Koh, Y. Oh and H.K. Lee, Nucl. Phys. **A530** (1991) 698
9) M. Rho, D.O. Riska and N.N. Scoccola, Phys. Lett. **B251** (1990) 597;
N.N. Scoccola, Nucl. Phys. A to be published
10) D.O. Riska and N.N. Scoccola, Phys. Lett. **B265** (1991) 188
11) G. Pari, B. Schwesinger and H. Walliser, Phys. Lett. **B255** (1991) 1

12) M. Rho, D.O. Riska and N.N. Scoccola, The Niels Bohr Institute preprint, Z. Phys. A in press
13) M. Praszalowicz, Phys. Lett. **B158** (1985) 264
14) H. Yabu and K. Ando, Nucl. Phys. **B301** (1988) 601
15) J. Gasser and H. Leutwyler, Ann. of Phys. **158** (1984) 142; Nucl. Phys. **B250** (1985) 465
16) M. Rho, Mod. Phys. Lett. A, to be published.
17) N.N. Scoccola and A. Wirzba, Phys. Lett. **B258** (1991) 451
18) G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. **B228** (1983) 552
19) G.S. Adkins and C.R. Nappi, Nucl. Phys. **B233** (1984) 109
20) C.A. Dominguez and N. Paver, Phys. Lett. **B197** (1987) 423
21) Particle Data Group, Phys. Lett. **B239** (1990) 1
22) A. De Rújula, H. Georgi and S.L. Glashow, Phys. Rev. **D12** (1975) 147
23) L.A. Copley, N. Isgur and G. Karl, Phys. Rev. **D20** (1979) 768;
K. Maltman and N. Isgur, Phys. Rev. **D22** (1980) 1701
24) A.L. Choudhury and V. Joshi, Phys. Rev. **D13** (1976) 3115
25) A.L. Choudhury and V. Joshi, Phys. Rev. **D13** (1976) 3120; **14** (E) (1976) 2437
26) D.B. Lichtenberg, Phys. Rev. **D15** (1977) 345
27) S.N. Jena and D.P. Rath, Phys. Rev. **D34** (1986) 196
28) S.K. Bose and L.P. Singh, Phys. Rev. **D22** (1980) 773
29) T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. **D12** (1975) 2060
30) Particle Data Group, Phys. Lett. **B204** (1988) 1
31) J. Franklin, D.B. Lichtenberg, W. Namgung and D. Carydas, Phys. Rev. **D24** (1981) 2910